The phases of cold magnetized quark matter in NJL-type models

N. N. Scoccola
Tandar Lab -CNEA– Buenos Aires

PLAN OF THE TALK

• Introduction
• Cold magnetized quark matter in a SU(2) NJL model with general flavor mixing and vector interactions.
• Extension to flavor SU(3)
• Color superconducting phases in the presence of an external magnetic field
• Outlook & Conclusions

Recently, there has been quite a lot of interest in investigating how the QCD phase diagram is affected by the presence of strong magnetic fields. Motivation: their possible existence in physically relevant situations:

High magnetic fields in non-central relativistic heavy ion collisions

**Motivation:**

Their possible existence in physically relevant situations:

Voloshin, QM2009

**Compact Stellar Objects:** Magnetars are estimated to have $B \sim 10^{14}-10^{15}$ G at the surface. It could be much higher in the interior (Duncan and Thompson (92/93))
Several theoretical/phenomenological questions arise:

- How does the QCD phase diagram look like when one includes a non-zero uniform B?
- Are there modifications in the nature of the phase transitions?
- Do chiral and deconfinement transitions behave differently?
- Which is the fate of the critical point(s)?
- ….

This has been investigated in a variety of approaches. For example [certainly incomplete list!]

- NJL and relatives (Klevansky, Lemmer (89); Klimenko et al. (92,..); Gusynin, Miransky, Shokovy (94/95); Ferrer, Incera et al (03..), Hiller, Osipov (07/08); Menezes et al (09); Fukushima, Ruggieri, Gatto (10) [PNJL]; …)

- $\chi$PT (Shushpanov, Smilga (97); Agasian, Shushpanov (00); Cohen, McGady, Werbos (07);…)

- Linear Sigma Model and MIT bag model: (Fraga, Mizher (08), Fraga, Palhares (12)…)

- Lattice QCD (D'Elia (10/11), Bali et al (11/12),…)

We consider a generalized form of the Nambu-Jona-Lasinio model which includes scalar-pseudoscalar, vector-axial-vector four-fermion couplings as well as t’Hooft determinant (instanton induced) interaction

\[ \mathcal{L} = \bar{\psi} (i \not{D} - \hat{m}) \psi + \mathcal{L}_{int} \]

where

\[
\mathcal{L}_{int} = G_1 \sum_{a=0}^{3} \left[ (\bar{\psi}_{a}\tau_{a}\psi)^2 + (\bar{\psi}_{a}\gamma_{5}\lambda_{a}\psi)^2 \right] - G_2 \sum_{a=0}^{3} \left[ (\bar{\psi}_{a}\gamma_{\mu}\tau_{a}\psi)^2 + (\bar{\psi}_{a}\gamma_{\mu}\gamma_{5}\tau_{a}\psi)^2 \right] \\
- G_3 \left[ (\bar{\psi}_{a}\gamma_{\mu}\psi)^2 + (\bar{\psi}_{a}\gamma_{\mu}\gamma_{5}\psi)^2 \right] - G_4 \left[ (\bar{\psi}_{a}\gamma_{\mu}\psi)^2 - (\bar{\psi}_{a}\gamma_{\mu}\gamma_{5}\psi)^2 \right] + 2G_D (d_+ + d_-)
\]

and \[ d_\pm = det[\bar{\psi}(1 \pm \gamma_5)\psi] \]

We work with the two lightest flavor \[ \psi^\dagger = (u, d) \]

[Note that Color and Dirac indexes are implicit]
The coupling of the quark fields to an external constant and homogenous magnetic field in the z-direction is done using minimal coupling i.e.

\[ \tilde{\partial} \rightarrow \tilde{\partial} - ie \tilde{A} \]

\[ \tilde{A} = \frac{B}{2}(-y, x, 0) \]

We will consider the situation of cold dense magnetized quark matter, i.e. we set \( T=0 \) and introduce one chemical potential for each flavor.

\[ \mu_f = \mu_u, \mu_d \]

Our calculations will be done in the Mean Field Approximation (MFA).

\[ (\bar{\psi} \mathcal{O} \psi)^2 = 2 < \bar{\psi} \mathcal{O} \psi > \bar{\psi} \mathcal{O} \psi \]

assuming that only the scalar and vector expectation values can be non-vanishing
As shown in Menezes et al, PRC79(09) the divergent vacuum contribution can be separated from the magnetic terms. The resulting thermodynamical potential reads

\[
\Omega^\text{MFA} = \sum_{f=u,d} P_f + \Omega_{\text{pot}}
\]

where

\[
P_f^{\text{vac}} = 4N_c \int \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + M_f^2},
\]

\[
P_f^{\text{mag}} = \frac{N_c}{2\pi^2} (|q_f|B)^2 \left[ \zeta^{(1,0)}(-1, x_f) - \frac{1}{2}(x_f^2 - x_f) \ln x_f + \frac{x_f^2}{4} \right],
\]

\[
P_f^{\text{med}} = \frac{N_c}{4\pi^2} |q_f|B \sum_{\nu=0}^{\nu_f^{\text{max}}} \alpha_{\nu} \left[ \tilde{\mu}_f \sqrt{\tilde{\mu}_f^2 - s_f(\nu, B)^2} - s_f(\nu, B)^2 \ln \left( \tilde{\mu}_f + \sqrt{\tilde{\mu}_f^2 - s_f(\nu, B)^2 s_f(\nu, B)} \right) \right]
\]

and

\[
M_f = m_c + \sigma_f ; \quad \tilde{\mu}_f = \mu_f - \omega_f \quad \sigma_f, \omega_f \text{ proportional to scalar and vector MF values, respectively}
\]

\[
x_f = \frac{M_f^2}{2|q_f|B} ; \quad s_f = \sqrt{M_f^2 + 2|q_f|B \nu} ; \quad \nu_f^{\text{max}} = \text{Floor} \left[ \left( \tilde{\mu}_f^2 - M_f^2 \right) / (2|q_f|B) \right]
\]

\[
\zeta(x, y) \quad \text{Riemann zeta}
\]
The remaining contribution to the thermodynamical potential is

$$\Omega_{pot} = \frac{(1 - c_s)(\sigma_u^2 + \sigma_d^2) - 2c_s \sigma_u \sigma_d}{8g_s(1 - 2c_s)} - \frac{(1 - c_v)(\omega_u^2 + \omega_d^2) - 2c_v \omega_u \omega_d}{8g_v(1 - 2c_v)}$$

where we have introduced the relevant combinations of coupling constants

$$g_s = G_1 + G_D \ ; \ \ ; \ \ \ g_v = G_2 + G_3 + G_4$$

$$c_s = \frac{G_D}{G_1 + G_D} \ ; \ \ ; \ \ \ c_v = \frac{G_3 + G_4}{2(G_2 + G_3 + G_4)}$$

$g_s$ and $g_v$ effective scalar and vector couplings, respectively.

$c_s$ regulates flavor mixing [$0 \leq c_s \leq \frac{1}{2}, \ \ \ c_s=\frac{1}{2}$ maximum flav.mixing (NJL)]

$c_v$ regulates isoscalar-isovector mixing [$0 \leq c_v \leq \frac{1}{2}, \ \ c_v=\frac{1}{2}$ pure isoscalar]

Model parameters will be set as it follows:

$m \ (=m_f)$, $g_s$ and $\Lambda$ chosen to reproduce the empirical values of $m_\pi$ and $f_\pi$ and a constituent quark mass in the phenomenological range $M_0 = 300 - 400$ MeV.

$g_v, \ c_s, \ c_v$ [phenomenological not well-known] will be taken as free parameters

Note: effective OGE indicates $g_v/g_s \sim \frac{1}{2}$. Also $\eta-\eta'$ mixing points to $c_s \sim 0.2$
We solve numerically the set of “gap” equations given by

\[ \frac{\partial \Omega_{MFA}(\sigma_u, \sigma_d, \omega_u, \omega_d)}{\partial \phi_i} = 0 \quad ; \quad \phi = (\sigma_u, \sigma_d, \omega_u, \omega_d) \]

to obtain \( \phi_i \) for given values of \( \mu_f \) and \( B \).

First order transitions are defined by a discontinuity of the order parameter \( \phi_i \). Crossover transitions are defined by the peak of the chiral susceptibilities

\[ \chi_{ch} = \frac{d < \bar{\psi}_f \psi_f >}{d m_f} \]

In what follows magnetic fields expressed in natural units

\[ eB = 1 \text{GeV}^2 \rightarrow B = 1.69 \times 10^{20} G \]
The «standard» NJL case (\(c_s = 1/2\), \(g_v = 0\))

\[M_0 = 340 \text{ MeV}\]

\[M_0 = 400 \text{ MeV}\]
Phase diagrams for different parametrizations

B (vacuum) phase: no LL is populated

Cₙ phase: up to n LL is populated but χSYM is broken (M is “large”)

Aₙ phase: up to n LL is populated but χSYM is restored (M is “small”)

Notation from: Ebert, Klimenko et al, PRD’99-NPA’03

Van Alphen-de Haas (vAdH) transitions (i.e transitions between A phases) are weak 1st order

Inverse Magnetic Catalysis (IMC): μₜ decreases for intermediate values of B

(Preis, Rebhan, Schmidt ’11)
M as a function of eB for various values $\mu$ of (set $M_0=320$ MeV)

Oscillations due to vAdH effect
The role of flavor mixing \( [g_v=0] \)

\[
C_n = n \\
A_n = \bar{n}
\]

For low \( c_s \)

u-quark and d-quark behave differently (different charges)

For \( c_s = 0.2 \) similar to full mixing (\( c_s = 0.5 \)
The role of vector interactions

Effect of $c_v$ very tiny

IMC effect decreases as $g_v/g_s$ increases

$c_s = 0.2$

$M_0 = 340$ MeV

$g_s/g_v = 0$

$g_s/g_v = 0.3$

$g_s/g_v = 0.5$

$M_0 = 400$ MeV

$g_s/g_v = 0$

$g_s/g_v = 0.3$

$g_s/g_v = 0.5$
The role of stellar matter conditions

Quark matter + e + \( \tilde{\mu} \)

\[ \beta \text{ Equilibrium} \]
\[ \mu_\mu = \mu_e \quad ; \quad \mu_d = \mu_u + \mu_e \]

Charge Neutrality
\[ \rho_e + \rho_\mu = \left( 2 \rho_u - \rho_d \right)/3 \]

IMC effect is further quenched

Pile-up of 1\textsuperscript{st} order transitions almost disappear as \( g_v/g_s \) increases
Extension to flavor SU(3)

Symmetric matter

Grunfeld, Menezes, Pinto, NNS, Phys.Rev. D90 (2014) 4, 044024

Stellar matter

\[ C_n = n \]
\[ A_n = \bar{n} \]

\[ \beta \text{ equilibrium} \quad \mu_d = \mu_s = \mu_u + \mu_e \quad ; \quad \mu_e = \mu_\mu \]

Charge neutrality \[ \rho_e + \rho_\mu = \frac{(2\rho_u - \rho_d - \rho_s)}{3} \]
Aspects of this problem in the context of NJL-like models have been already discussed in Ferrer, de la Incera ’05, Fukushima, Warringa’08, Noronha, Shovkovy’07, Fayazbakhsh, Sadooghi’10, etc. Here, we try to construct phase diagram. For simplicity we only consider the simplest NJL model with additional 2SC interactions

\[ \mathcal{L}_{\text{int}} = G \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2 \right] + H \left[ (i\bar{\psi}^C \epsilon_f \epsilon_c^3 \gamma_5\psi)(i\bar{\psi} \epsilon_f \epsilon_c^3 \gamma_5\psi^C) \right] \]

Typically, \( H/G \sim 0.75 \). \( \psi^C = i \gamma_2 \gamma_0 \bar{\psi}^T \) and \( \epsilon_f \) and \( \epsilon_c^3 \) are antisymmetric tensors in flavor and color space, respectively. Color/charge neutrality and \( \beta \) equilibrium not considered for simplicity.

In the presence of 2SC gap \( \Delta \) the photon acquires finite mass (Meissner effect). However (Alford et al NPB571(00)) there is a combination of photon and 8th gluon field that remains massless. The corresponding rotated charges are

\[
\begin{align*}
\mathbf{u}_r & \quad \mathbf{u}_g & \quad \mathbf{u}_b & \quad \mathbf{d}_r & \quad \mathbf{d}_g & \quad \mathbf{d}_b \\
1/2 & \quad 1/2 & \quad 1 & \quad -1/2 & \quad -1/2 & \quad 0
\end{align*}
\]

The rotated electron charge is \( \tilde{e} = e \cos \theta \) with \( \theta \sim 1/20 \) [Gorbar PRD62(00)].
The MFA thermodynamical potential reads

\[ \Omega = \frac{(M - m)^2}{4G} + \frac{\Delta^2}{4H} - \sum_{|\tilde{q}| = 0, \frac{1}{2}, 1} P_{|\tilde{q}|} \]

where

\[
P_{|\tilde{q}| = 0} = \int \frac{d^3p}{(2\pi)^3} \sum_{s = \pm} |E_0^s| \]

\[
P_{|\tilde{q}| = \frac{1}{2}, 1} = \frac{\tilde{e}B}{4\pi^2} \sum_{k=0}^{\infty} \alpha_k \int dp_z \sum_{s = \pm} |E_{|\tilde{q}|}^s| \]

Obviously, regularization is required. In the literature this was commonly done by introducing a regulator function \( h(q) \) in the integrands. Here, \( q = p \) in \( P_0 \) and \( q = [p_z^2 + 2k|\tilde{q}|B]^{1/2} \) in \( P_{1/2, 1} \). While this might be suitable for certain applications, in general, it can problematic.

Sharp regulator

\[ h(q) = \theta(q - \Lambda) \]

Leads to strong oscillations

Softer regulator, like e.g.

\[ h(q) = \left(1 + \exp\left(\frac{(q / \Lambda - 1)}{a}\right)\right)^{-1} \quad ; \quad a = 0.05 \]

improves the situation but not completely solved
We propose an alternative regularization method. In particular we find that, by summing and subtracting convenient terms, $P_{1/2}$ can be cast into the form

$$P_{|\tilde{q}|=1/2} = \frac{2}{\pi^2} \int_0^\Lambda dp \ p^2 \ (E^+_{\Delta} + E^-_{\Delta}) + \frac{(\tilde{e}B)^2}{2\pi^2} \left[ \xi^{(1,0)}(-1, y) + \frac{y - y^2}{2} \ln y + \frac{y^2}{4} \right]$$

$$+ \frac{(\tilde{e}B)^2}{2\pi^2} \int_0^\infty dp \ \left[ \sum_{k=0}^{\infty} \alpha_k \ f(p^2 + k) \right] - 2 \int_0^\infty dx \ f(p^2 + x)$$

where

$$f(z) = \left[ \sum_{s=\pm 1} \sqrt{(\sqrt{z} + x + s\mu/\sqrt{\tilde{e}B})^2 + y - x} \right] - 2\sqrt{z + y}.$$ 

$$E^\pm_{\Delta} = \sqrt{(\sqrt{p^2 + M^2 \pm \mu})^2 + \Delta^2}; \ y = (M^2 + \Delta^2)/(\tilde{e}B); \ x = M^2/(\tilde{e}B)$$

Note that only the first term (which is B-independent) requires regularization: we use 3D cutoff. Last term can be proven to be finite. This regularization method can be considered as an extension of that of Menezes et al, PRC70(09) that we, accordingly, use for $P_1$. We call this regularization method “Magnetic Field Independent regularization” (MFIR)
Lowest vAdH turns into a COv. Eventually disappears as H/G increases.

Only one transition at low B for H/G=0.75 and Set M_0=340.

For H/G=1, low B the lowest transition is 2^{nd} order. Note the existence of a new phase: D (Coexistence of M and \Delta).
Phase Diagram

Order parameters as functions of $eB$ for various $\mu$

- $H/G = 0.5$
- $H/G = 0.75$
- $H/G = 1.0$
Summary and Outlook

• Within NJL-type models the detailed form of the phase diagram of the cold magnetized quark matter is rather different depending on the parameterization used. In fact, there can be a quite rich structure due to the subsequent population of the Landau levels as $\mu$ increases.

• For arbitrary (small) flavor mixing (FM) the phase diagram can be rather complex. For phenomenological acceptable values of FM ($c_s \sim 0.2$) results similar to maximum flavor mixing.

• Vector mesons as well as charge neutrality and beta equilibrium conditions tend to make the IMC effect weaker (or even disappear).

• A method to completely avoid unphysical oscillations when Color Superconductive channels are taken into account has been introduced. Physical oscillations (Fukushima, Warringa’08, Noronha, Shovkovy’07) seen for small values of H/G.

• Future: Incorporation of compact stars conditions and strangeness in the presence of CSC phases….
The role of the regularization in the behaviour of the condensate \( a \mu = T = 0 \)

\[
\Delta \Sigma(B, T) = \frac{2m_o}{(f \pi m_{\pi})^2} \left( <\bar{f}f>_{B,T} - <\bar{f}f>_{0,0} \right) + 1
\]

where

Blue line: (P)NJL taken from Ruggieri et al’11.

Form factor regularization Lor 5 used.
Form Factor regularization (Lor5) for different model parametrizations.

Both lattice and NJL predict almost linear behaviour.

However, slope $\gamma$ quite different.

$\gamma^{\text{Lat}} \approx 1.32 \, \text{GeV}^{-2}$

$\gamma^{\text{FF}} \approx 3.55 - 3.65 \, \text{GeV}^{-2}$

MFIR for different model parametrizations.

Also predicts almost linear bahaviour.

However,

$\gamma^{\text{MFIR}} \approx 1.38 - 1.49 \, \text{GeV}^{-2}$

not far from lattice prediction
Difference $\Sigma_u - \Sigma_d$ using MFIR for different model parametrizations (blue) as compared with lattice results of Bali et al (dots)
Backup material
A possible solution to this problem has been suggested in the framework of the EPNJL (Ferreira, Costa, Menezes, Providencia, NNS PRD(14)). We proposed that PL potential depends on B through the $T_0$ parameter

$$T_0(eB) = T_0(eB = 0) + \xi(eB)^2 + \eta(eB)^4$$

Condensates as functions of eB for various $T$

$L_0(eB=0) = 270$ MeV
Finite quark masses

Sets de parámetros:

Mass

Chiral susceptibility

\( \mu \) [MeV]

\( eB \) [GeV^2]

\( A_1 \)

\( A_2 \)

\( A_3 \)

\( B \)

\( C_0 \)

\( C_1 \)

\( C_2 \)
Diagramas de fase para varios B

Set A

Set B

Dotted Deconf. Dashed Chiral $T_0=208$ MeV