A primer on Gravitational Lensing

Eduardo S. Cypriano\textsuperscript{1}

\textsuperscript{1}Departamento de Astronomia
Instituto de Astronomia, Geofísica e Ciências Atmosféricas
Universidade de São Paulo

IFT School on Dark Mater, 2016
Outline

Gravitational Lensing basics

Micro lensing

Strong lensing

Weak Lensing
Gravitational lensing geometry

- Angular diameter distances: $D_s$, $D_d$ and $D_{ds}$
- Source position: $\beta = \eta / D_s$ - Impact parameter
- Image position: $\theta = \xi / D_d$
- Deflexion angle: $\alpha = \hat{\alpha} \cdot D_{ds} / D_s$

Figure from Narayan & Bartelmann (1996; arXiv:astro-ph/9606001)
The Lensing Equation

\[ \eta = \frac{D_s}{D_d} \xi - D_{ds} \hat{\alpha}(\xi) \]
The Lensing Equation

\[ \eta = \frac{D_s}{D_d} \xi - D_{ds} \hat{\alpha}(\xi) \]

Using the angular quantities instead of the physical ones we get:

\[ \beta = \theta - \frac{D_{ds}}{D_s} \hat{\alpha}(D_{d}\theta) \]
The Lensing Equation

\[ \eta = \frac{D_s}{D_d} \xi - D_{ds} \hat{\alpha}(\xi) \]

Using the angular quantities instead of the physical ones we get:

\[ \beta = \theta - \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \theta) \]

Then by using the physical deflexion angle we arrive to the **Lens Equation**:

\[ \beta = \theta - \alpha(\theta) \]
The deflection angle

- From GR one can estimate the deflection angle due to a mass point:
  \[ \hat{\alpha} = \frac{4 \, G \, M}{c^2 \xi} \]

- The projected angle is then:
  \[ \alpha = |\alpha| = \frac{D_{ds} \, 4 \, G \, M}{D_s \, c^2 D_d |\theta|} \]

- Considering now a direction, as all the angles lies in the image-lens direction, we have:
  \[ \alpha = \frac{D_{ds} \, 4 \, G \, M \, \theta}{D_s \, c^2 D_d |\theta| \, |\theta|} \]
The deflection angle

- This prediction lead to the first observational proof of the General Relativity

ANNOUNCEMENT OF EDDINGTON’S DISCOVERY. CREDIT: ILLUSTRATED LONDON NEWS (1919)
The lensing effect

- Gravitational lensing causes several effects on the images of the sources:
  1. Radial displacement
The lensing effect

- Gravitational lensing causes several effects on the images of the sources:
  1. Radial displacement
  2. Multiple imaging (given certain conditions)
The lensing effect

- Gravitational lensing causes several effects on the images of the sources:
  1. Radial displacement
  2. Multiple imaging (given certain conditions)
  3. Magnification of the angular size
The lensing effect

- Gravitational lensing causes several effects on the images of the sources:
  1. Radial displacement
  2. Multiple imaging (given certain conditions)
  3. Magnification of the angular size

经济效益 conservation

- and flux

The prevalence/interest of one or more of those effects over the others have to do with the lensing regime.
The lensing effect

- Gravitational lensing causes several effects on the images of the sources:
  1. Radial displacement
  2. Multiple imaging (given certain conditions)
  3. Magnification of the angular size
     \[ \text{surface brightness conservation} \quad \rightarrow \] and flux
  4. Distortion
The lensing effect

- Gravitational lensing causes several effects on the images of the sources:
  1. Radial displacement
  2. Multiple imaging (given certain conditions)
  3. Magnification of the angular size
     \[\text{surface brightness conservation}\] and flux
  4. Distortion
  5. Time Delay

- The prevalence/interest of one or more of those effects over the others have to do with the lensing regime.
Lensing regimes

- Different lensing regimes differentiate themselves by:
  1. The lens mass distribution
  2. The distances involved
  3. The impact parameter
Lensing regimes

- Different lensing regimes differentiate themselves by:
  1. The lens mass distribution
  2. The distances involved
  3. The impact parameter

- The main lensing regimes are:
  1. Micro lensing
  2. Strong lensing
  3. Weak lensing

▶ Different lensing regimes differentiate themselves by:
  1. The lens mass distribution
  2. The distances involved
  3. The impact parameter

▶ The main lensing regimes are:
  1. Micro lensing
  2. Strong lensing
  3. Weak lensing
Micro lensing

1. The lens mass distribution
2. The distances involved
3. The impact parameter
Micro lensing

1. The lens mass distribution: Compact, “point mass”
2. The distances involved
3. The impact parameter
Micro lensing

1. The lens mass distribution: Compact, “point mass”
2. The distances involved: $\mathcal{O} \sim$ tenths of kpc
3. The impact parameter
Micro lensing

1. The lens mass distribution: Compact, “point mass”
2. The distances involved: $O \sim$ tenths of kpc
3. The impact parameter: variable with very small minimum
The point mass lens

- The lens equation can be easily solved in this case.
- Let's first define a 'natural scale', the *Einstein angle* for this lens:

\[ \theta_E = \sqrt{\frac{4GM}{c^2 \frac{D_{ds}}{D_s D_d}}} \]
The point mass lens

- The lens equation can be easily solved in this case.
- Let's first define a ‘natural scale’, the *Einstein angle* for this lens:

\[
\theta_E = \sqrt{\frac{4GM}{c^2}} \frac{D_{ds}}{D_s D_d}
\]

- The lens equation then becomes:

\[
\beta = \theta - \frac{\theta_E^2}{\theta}
\]
The point mass lens

- The lens equation can be easily solved in this case.
- Let's first define a 'natural scale', the Einstein angle for this lens:

\[ \theta_E = \sqrt{\frac{4GM}{c^2}} \frac{D_{ds}}{D_s D_d} \]

- The lens equation then becomes:

\[ \beta = \theta - \frac{\theta_E^2}{\theta} \]

- It has two solutions:

\[ \theta_{\pm} = \frac{1}{2} \left( \beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right) \]
The point mass lens

• What is ‘natural’ about the *Einstein angle or radius*?

• If $\beta = 0 \rightarrow \theta = \theta_E$
The point mass lens

- What is ‘natural’ about the *Einstein angle or radius*?

- If $\beta = 0 \rightarrow \theta = \theta_E$
The point mass lens

- What is ‘natural’ about the *Einstein angle or radius*?

- If $\beta = 0 \rightarrow \theta = \theta_E$

- The distance between two images (solutions) is $2\theta_E$
The point mass lens

- What is ‘natural’ about the *Einstein angle or radius*?

- If $\beta = 0 \rightarrow \theta = \theta_E$

- The distance between two images (solutions) is $2\theta_E$

- The density inside the Einstein radius is the critical lensing density

\[ \Sigma_{cr} = \frac{c^2}{4\pi G} \frac{Ds}{D_d D_{ds}} \]

above which multiple imaging occur.
The point mass lens

Solution

\[ \theta_{\pm} = \frac{1}{2} \left( \beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right) \]

\[ x = \frac{\theta}{\theta_E}; y = \frac{\beta}{\theta_E} \rightarrow x_{\pm} = \frac{1}{2} \left( y \pm \sqrt{y + 4} \right) \]

Point source

Figura de Narayan & Bartelmann

The point mass lens

Solution

$$\theta_{\pm} = \frac{1}{2} \left( \beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right) \quad \text{with} \quad x = \frac{x}{\theta_E}; \quad y = \frac{\beta}{\theta_E} \quad \Rightarrow \quad x_{\pm} = \frac{1}{2} \left( y \pm \sqrt{y + 4} \right)$$

Point source


Extended source

Magnification

- The flux magnification \( q \) is equal to the ration of the image and source areas.
- For a circularly symmetric lens:

\[
\mu = \frac{\theta}{\beta} \frac{d\theta}{d\beta}
\]
Magnification

- The flux magnification $q$ is equal to the ratio of the image and source areas.
- For a circularly symmetric lens:
  \[ \mu = \frac{\theta}{\beta} \frac{d\theta}{d\beta} \]
- For the point lens, then
  \[ \mu_\pm = \left[ 1 - \left( \frac{\theta_E}{\theta_\pm} \right)^4 \right] \]
Magnification

- The flux magnification $q$ is equal to the ratio of the image and source areas.
- For a circularly symmetric lens:

$$
\mu = \frac{\theta}{\beta} \frac{d\theta}{d\beta}
$$

- For the point lens, then

$$
\mu_{\pm} = \left[ 1 - \left( \frac{\theta_E}{\theta_{\pm}} \right)^4 \right]
$$

, 

- and

$$
\mu = |\mu_+| + |\mu_-| = \frac{y^2 + 2}{y\sqrt{y^2 + 4}}
$$
Micro lenses

- In nature the micro lenses are usually stars, planets and other compact objects in or close to the Galaxy.
- Due to its proximity the proper motion of those objects is relevant.
Micro lenses

- In nature the micro lenses are usually stars, planets and other compact objects in or close to the Galaxy.
- Due to its proximity the proper motion of those objects is relevant.

Figura de Schneider 2006
Micro lenses

Typical Einstein radius

\[ \theta_E = 0.902 \text{mas} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{D_d}{10 \text{kpc}} \right)^{-1/2} \left( 1 - \frac{D_d}{D_s} \right)^{-1/2} \]
Micro lenses

▶ Typical *Einstein radius*

\[ \theta_E = 0.902 \text{mas} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{D_d}{10 \text{kpc}} \right)^{-1/2} \left( 1 - \frac{D_d}{D_s} \right)^{-1/2} \]

▶ We cannot observationally resolve the multiple imaging, but we can measure the variation of the flux due to the magnification.

\[ \theta_E \] refers to the Einstein radius, which is a characteristic size of the gravitational lensing effect. The formula above shows how the Einstein radius depends on the mass of the lensing object, the distance to the lens, and the source. The Einstein radius is crucial in understanding the dynamics of the lensing effect.
Micro lenses

- Typical *Einstein radius*

\[ \theta_E = 0.902 \text{mas} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{D_d}{10 \text{kpc}} \right)^{-1/2} \left( 1 - \frac{D_d}{D_s} \right)^{-1/2} \]

- We cannot observationally resolve the multiple imaging, but we can measure the variation of the flux due to the magnification.

Note that microlensing should produce symmetric and achromatic light curves as above.
Astrophysical Applications of Microlensing

1. Extra solar planet detection

OGLE-2005-BLG-390; Credit: ESO
Astrophysical Applications of Microlensing

1. **Extra solar planet detection**

   OGLE-2005-BLG-390; Credit: ESO

2. **Stellar & Galactic astrophysics**
Astrophysical Applications of Microlensing

1. Extra solar planet detection

OGLE-2005-BLG-390; Credit: ESO

2. Stellar & Galactic astrophysics

3. Quantification of the MAssive Compact Halo Objects (MACHOs) in the Galaxy.
MACHOs in the Galaxy

- Two main projects: MACHO & EROS: Observation of the LMC & SMC

Figure from Alcock et al. 2000
MACHOs in the Galaxy

- Macho project: 5.7 years; LMC.
MACHOs in the Galaxy

- Macho project: 5.7 years; LMC. → 13-17 events; 100% MACHO halo ruled out with 95% c.l.
MACHOs in the Galaxy

- Macho project: 5.7 years; LMC. → 13-17 events; 100% MACHO halo ruled out with 95% c.l.
- EROS project: 5 years; SMC

Figure from Afonso et al. 2003
MACHOs in the Galaxy

- Macho project: 5.7 years; LMC $\rightarrow$ 13-17 events; 100% MACHO halo ruled out with 95% c.l.
- EROS project: 5 years; SMC $\rightarrow$ 5 events; 25% of less of the MW halo is composed by MACHOs (95% c.l.)

Figure from Afonso et al. 2003
Microlensing surveys, in practice, consolidate the notion that dark matter must be constituted by some kind of (non-baryonic) particles, instead of microscopic objects.
Strong lensing

1. The lens mass distribution
2. The distances involved
3. The impact parameter
Strong lensing

1. The lens mass distribution: extended - galaxies and galaxy clusters
2. The distances involved
3. The impact parameter
Strong lensing

1. The lens mass distribution: extended - galaxies and galaxy clusters
2. The distances involved: $O \sim \text{Gpc}$
3. The impact parameter
Strong lensing

1. The lens mass distribution: extended - galaxies and galaxy clusters
2. The distances involved: $\mathcal{O} \sim \text{Gpc}$
3. The impact parameter: $\leq \theta_E \rightarrow \text{multiple imaging}$
In practice the lens dimensions are much smaller than the other distances involved.
Extendend lenses

- In practice the lens dimensions are much smaller than the other distances involved.
- In this limit the *thin lens approximation* is valid:
In practice the lens dimensions are much smaller than the other distances involved.

In this limit the *thin lens approximation* is valid: The relevant quantity is not the 3D mass distribution ($\rho$) but instead the projected (2D) mass density ($\Sigma$)
The deflection angle can then be calculated as the sum of the contributions of every mass element:

\[ \hat{\alpha} = \sum_i \frac{4 \, G \, m_i}{c^2} \frac{\xi - \xi_i}{|\xi - \xi_i|^2} \]
Extendend lenses

- The deflection angle can then be calculated as the sum of the contributions of every mass element:

\[ \hat{\alpha} = \sum_i \frac{4G m_i}{c^2} \frac{\xi - \xi_i}{|\xi - \xi_i|^2} \]

- For a continuous distribution mass distribution:

\[ \hat{\alpha}(\xi) = \frac{4G}{c^2} \int \Sigma(\xi') \frac{\xi - \xi'}{|\xi - \xi'|^2} \ d^2\xi' \]
Lens models

- Singular Isothermal Sphere (SIS)

\[ \rho(r) = \frac{\sigma_v^2}{2\pi Gr^2} \]
Lens models

▶ Singular Isothermal Sphere (SIS)

\[ \rho(r) = \frac{\sigma_v^2}{2\pi Gr^2} \]

- Elliptical galaxies
Lens models

- Singular Isothermal Sphere (SIS)
  \[ \rho(r) = \frac{\sigma_v^2}{2\pi Gr^2} \]
  - Elliptical galaxies
- Navarro, Frenk & White (NFW)
  \[ \rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \]
Lens models

- Singular Isothermal Sphere (SIS)

\[ \rho(r) = \frac{\sigma_v^2}{2\pi Gr^2} \]

- Elliptical galaxies

- Navarro, Frenk & White (NFW)

\[ \rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \]

- Clusters of Galaxies
Lens models

- Singular Isothermal Sphere (SIS)
  \[ \rho(r) = \frac{\sigma_v^2}{2\pi Gr^2} \]
- Elliptical galaxies

- Navarro, Frenk & White (NFW)
  \[ \rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \]
- Clusters of Galaxies
  \[ \rho(r < r_s) \propto r^{-1}; \rho(r \sim r_s) \propto r^{-2}; \rho(r > r_s) \propto r^{-3} \]
Strong Lensing Modelling

- Given a lens mass model and the position of the images we can solve the lens equation

\[ \beta = \theta - \alpha(\theta) \]

- This is a mapping problem:
Strong Lensing Modelling

- Given a lens mass model and the position of the images we can solve the lens equation

\[ \beta = \theta - \alpha(\theta) \]

- This is a mapping problem:

\( I_i = I_0 \)

\( I_x \)

\( I_u \)

\( \beta \)

\( \theta \)

\( \alpha(\theta) \)
Strong Lensing Modelling

Circular Lens

Source Plane

Image Plane

Figure from Hattori, Kneib & Makino, arXiv:astro-ph/9905009
Strong Lensing Modelling

Circular Lens
Strong Lensing Modelling

Elliptical Lens

Source Plane

Image Plane

Figure from Hattori, Kneib & Makino, arXiv:astro-ph/9905009
Strong Lensing Modelling

Elliptical Lens
Giants arcs in Galaxy Clusters

Results

1. Reinforcement that clusters are dark matter dominated ($M/L \sim 100 - 300 M_{\odot}/L_{\odot}$)
Giants arcs in Galaxy Clusters

Results

1. Reinforcement that clusters are dark matter dominated \((M/L \sim 100 - 300M_\odot/L_\odot)\)
2. Arcs are large and relatively regular
Giants arcs in Galaxy Clusters

Results

1. Reinforcement that clusters are dark matter dominated \( (M/L \sim 100 - 300M_\odot/L_\odot) \)

2. Arcs are large and relatively regular \( \rightarrow \) The dark matter is smoothly distributed instead of attached to galaxies.
Giants arcs in Galaxy Clusters

Results

1. Reinforcement that clusters are dark matter dominated ($M/L \sim 100 - 300 M_\odot/L_\odot$).
2. The dark matter is smoothly distributed instead of attached to galaxies.
Giants arcs in Galaxy Clusters

Results

1. Reinforcement that clusters are dark matter dominated \((M/L \sim 100 - 300 M_{\odot}/L_{\odot})\).
2. The dark matter is smoothly distributed instead of attached to galaxies.
3. Strong lensing clusters are not symmetrical and often substructured.
Giants arcs in Galaxy Clusters

Results

1. Reinforcement that clusters are dark matter dominated ($M/L \sim 100 - 300 M_\odot/L_\odot$).
2. The dark matter is smoothly distributed instead of attached to galaxies.
3. Strong lensing clusters are not symmetrical and often substructured.
4. The slope of the mass profile at the very centre is shallower than numerical predictions.
Galaxy strong lensing

SLACS: The Sloan Lens ACS Survey
A. Bolton (U. Hawai’i IfA), L. Koopmans (Kapteyn), T. Treu (UCSB), R. Gavazzi (IAP Paris), L. Moustakas (JPL/Caltech), S. Burles (MIT)

www.SLACS.org

Image credit: A. Bolton, for the SLACS team and NASA/ESA

Figure from Bolton et al. (2008)
1. Dark matter is about 1/4 of the mass at the half-light radius (70% at 5 $r_e$).
Galaxy strong lensing
Results for early-type galaxies

1. Dark matter is about 1/4 of the mass at the half-light radius (70% at 5 \( r_e \)).
2. The mass profile is isothermal (\( \rho \propto r^{-2} \)) with very little scatter.
Weak lensing

1. The lens mass distribution
2. The distances involved
3. The impact parameter
Weak lensing

1. The lens mass distribution: extended - galaxies, clusters and the large scale structure
2. The distances involved
3. The impact parameter
Weak lensing

1. The lens mass distribution: extended - galaxies, clusters and the large scale structure
2. The distances involved: $O \sim \text{Gpc}$
3. The impact parameter
Weak lensing

1. The lens mass distribution: extended - galaxies, clusters and the large scale structure
2. The distances involved: $O \sim \text{Gpc}$
3. The impact parameter: $\gg \theta_E$
Gravitational Image Distortion

The gravitational lensing distorts the images

- Convergence alone
- Convergence + Shear

Gravitational Lensing

Source → Image
Gravitational Image Distortion

- The gravitational lensing distorts the images

![Diagram showing gravitational lensing with source, gravitational lensing, convergence alone, and convergence + shear]

- The isotropic distortion is associated with the convergence: \( \kappa = \Sigma(r)/\Sigma_{cr} \)
  and anisotropic with the shear: \( \gamma_t \times \Sigma_{cr} = \Sigma(r) - \overline{\Sigma(r)} \)
The gravitational lensing distorts the images

The isotropic distortion is associated with the convergence: \( \kappa = \sum(r)/\sum cr \)
and anisotropic with the shear: \( \gamma_t \times \sum cr = \sum(r) - \overline{\sum(r)} \)
Both \( \kappa \) and \( \gamma \) are second derivatives of the lensing effective potential.
Given the large impact parameter the distortion is very small and is dominated by the galaxies own shapes.
Given the large impact parameter the distortion is very small and is dominated by the galaxies own shapes.

The weak shear can be detected statistically, however, by using samples with large numbers of galaxies:

$$\langle e \rangle = \gamma/(1 - \kappa) \simeq \gamma$$
Given the large impact parameter the distortion is very small and is dominated by the galaxies own shapes.

The weak shear can be detected statistically, however, by using samples with large numbers of galaxies:
\[
\langle e \rangle = \gamma / (1 - \kappa) \simeq \gamma
\]

Allow mass measurements far beyond the strong lensing area.
Weak-lensing measurements are quite delicate as one has to measure shapes of small source galaxies. Figure from Bridle et al. (2008; arXiv:0802.1214)
Weak lensing mass maps

- The grav. shear field can be used to reconstruct the projected mass distribution

**Weak gravitational lensing: patterns in a shear field**

![Image showing patterns in a shear field](image-url)
The Bullet Cluster

- On the merging cluster 1E 0657-56 gravity does not follow the baryonic mass (mostly gas)

One of the most convincing evidences of the existence of dark matter.
Cluster Mass Profiles

- General high quality results can be obtained by stacking the weak lensing signal of samples of lenses.

Figure from Okabe et al. (2010)
Cluster Mass Profiles

- General high quality results can be obtained by stacking the weak lensing signal of samples of lenses.

![Cluster Mass Profile Graph](image)

Figure from Okabe et al. (2010)

- The cluster mass profile is curved in a log-log space and consistent with numerical simulations such as NFW.
Mass measurements

- Weak grav. lensing cluster mass measurements are regarded as the most trustworthy as they do not suffer from the hydrostatic mass bias.
- They are ideal to calibrate mass-observable relations.
Mass measurements

- Those scale relations are crucial for cluster-count based cosmology. Constrains in the (dark) matter density and clumpiness

Figure from Mantz et al. (2014)
Mass measurements

- Those scale relations are crucial for cluster-count based cosmology. Constrains in the (dark) matter density and clumpiness

![Graph showing mass measurements](image)

- With a larger redshift range constrains on dark energy as well.

Figure from Mantz et al. (2014)
Cosmic shear

- Weak lensing by the large scale structure.

Figures from http://www.cfht.hawaii.edu/News/Lensing/
Cosmic shear

- The signal is even smaller than the cluster weak lensing
- Measurements uses the 2-point correlation of shapes.

Figure from Kaiser et al. (2000; arXiv:astro-ph/0003338)
Cosmic shear

- Similar sensitive to cosmological parameters as cluster-counts.

Figure from Heymans et al. (2013)

- Cosmics shear and Cluster counts have the potential to discriminate between dark energy and modified gravitation.