Study of thermo-magnetic hadron matter using the local Nambu-Jona-Lasinio model and its compatibility with Lattice QCD.

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May 12, 2016
Outline

1. QCD
2. NJL
3. (Inverse) Magnetic Catalysis
4. NJL with magnetic fields
5. Summary
We have a quark wave function \((\psi_\alpha)^a_f\), where \(\alpha\) denotes spin, \(a\) colour and \(f\) flavour.

A gluon field \(A^a_\mu\)

We have the QCD lagrangian

\[
\mathcal{L} = \bar{\psi} \left( i \slashed{D} + m_f \right) \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu},
\]

where

\[
\begin{align*}
i \slashed{D} &= \gamma^\mu \left( i \partial_\mu + g s A^a_\mu T^a \right) \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g s f^{abc} A^b_\mu A^c_\nu.
\end{align*}
\]
Features of QCD

- Asymptotic freedom

\[ \alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}) \ln \left( \frac{Q^2}{\Lambda^2} \right)} \]

- Confinement

- Chiral Symmetry
Nambu Jona Lasinio model

- Simplified model of strong interactions (1961).
- Superconductivity mechanism $\simeq$ strong interactions.

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + G \left[ (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 \right]$$

- NJL interactions respect the symmetries

$$\mathcal{S}_{NJL} = SU(3)_c \otimes SU(N_f)_A \otimes SU(N_f)_V \otimes SU(N_f)_A \otimes C \otimes P \otimes T$$
Se symmetry is respected for

\[ \psi \rightarrow e^{-it \cdot \theta \mathbf{v}} \psi \]
\[ \psi \rightarrow e^{-i\gamma_5 t \cdot \theta \mathbf{v}} \psi \]
\[ \psi \rightarrow e^{-i\theta} \psi \]
\[ \psi \rightarrow e^{-i\gamma_5 \theta} \psi \]

NJL is non-renormalizable: it needs an ultraviolet cutoff.

By the use of an infrared cutoff, it is possible to simulate confinement.
• The most general $N_f = 2$ NJL Lagrangian that respect the symmetries is

$$\mathcal{L} = \bar{\psi} (i \slashed{\partial} - m) \psi + \frac{G_\pi}{2} \left[ (\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \tau \psi)^2 \right] - \frac{G_\omega}{2} (\bar{\psi} \gamma^\mu \psi)^2$$

$$- \frac{G_\rho}{2} \left[ (\bar{\psi} \gamma^\mu \tau \psi)^2 + (\bar{\psi} \gamma_5 \tau \psi)^2 \right] + \frac{G_f}{2} (\bar{\psi} \gamma^\mu \gamma_5 \psi)^2$$

$$+ G_\eta \left[ (\bar{\psi} \gamma_5 \tau \psi)^2 - (\bar{\psi} \tau \psi)^2 \right] + G_T \left[ (\bar{\psi} i \sigma^{\mu\nu} \psi)^2 - (\bar{\psi} i \sigma^{\mu\nu} \tau \psi)^2 \right].$$

• $\mathcal{L}$ is $U(1)_A$ invariant if $G_\pi = -G_\eta$ and $G_T = 0$.

• The most general $N_f = 3$ NJL Lagrangian that respect the symmetries is

$$\mathcal{L}_I = G_\pi \left[ \frac{1}{6} (\bar{\psi} \psi)^2 - \frac{1}{6} (\bar{\psi} \gamma_5 \tau \psi)^2 + (\bar{\psi} \tau \psi)^2 - (\bar{\psi} \gamma_5 \tau \psi)^2 \right]$$

$$- \frac{G_\omega}{2} (\bar{\psi} \gamma^\mu \psi)^2 - \frac{G_\rho}{2} \left[ (\bar{\psi} \gamma^\mu \tau \psi)^2 + (\bar{\psi} \gamma_5 \tau \psi)^2 \right]$$

$$- \frac{G_f}{2} (\bar{\psi} \gamma^\mu \gamma_5 \psi)^2.$$
Enlarging the flavour space, reduce the number of coupling from six to four.

NJL gap equation

\[ S^{-1}(k) = S_0^{-1}(k) - \Sigma(k) = (\not{k} - m) - 2G \int \frac{d^4 p}{(2\pi)^4} \text{Tr} [iS(p)] \, . \]

For a propagator of the form

\[ S^{-1}(k) = \not{k} - M + i\epsilon, \]

The constituent mass satisfies the equation

\[ M = m + M \frac{3G}{\pi^2} \int d\tau \frac{e^{-\tau M^2}}{\tau^2} \, . \]

For \( G > G_{cr} \), the lowest energy has dynamical symmetry breaking
The poles of the scattering T matrix quark-antiquark, contains poles of the bound states, which corresponds to the physical mesons.

The scattering matrix is solved by the Bethe-Salpeter equation

\[
T_{i}^{\alpha\beta,\gamma\delta} = (\gamma_{5}\tau_{i})_{\alpha\beta} \frac{-2iG}{1 + 2G \Pi(q^{2})} (\gamma_{5}\tau_{i})_{\gamma\delta}
\]

The pion T matrix is
The pion mass is given by the condition \(1 + 2G\Pi(q^2 = m_{\pi}^2) = 0\).

Since

\[
\Pi(q^2) = \frac{m}{2GM} - \frac{1}{2G} - q^2 I(q^2),
\]

we have

\[
m_{\pi}^2 = \frac{m}{2GM I(m_{\pi}^2)}.
\]

So, \(m \to 0\), then \(m_{\pi} \to 0\).

Goldberger Treiman and Gell-Mann-Oakes-Renner (GMOR) are also satisfied.
(Inverse) Magnetic Catalysis

Figure:

Juan Cristóbal Rojas (VFU) Study of thermo-magnetic hadron matter using the local Nambu-Jona-Lasinio model and its compatibility with Lattice QCD. May 12, 2016 13 / 31
Dirac Fermions in 2+1 D

\[
\langle \bar{\psi} \psi \rangle \approx \frac{|eB|}{2\pi} \text{sign}(m_0) - \frac{m_0}{\pi^{3/2}} \left[ \Lambda + \sqrt{\frac{\pi}{2}} \frac{|eB|}{2} \zeta \left( \frac{1}{2}, 1 + \frac{m_0^2}{2 \cdot |eB|} \right) \right].
\]

NJL model in 2+1 D

\[
|m| \approx G \frac{|eB|}{2\pi}.
\]

NJL in 3+1 D at weak coupling

\[
\langle \bar{\psi} \psi \rangle \approx -\frac{m_0}{(2\pi)^2} \left[ \Lambda^2 + |eB| \ln \frac{|eB|}{\pi m_0^2} - m_0^2 \ln \frac{\chi^2}{2 \cdot |eB|} + \cdots \right].
\]
• NJL in 3+1 D, with mean field approximation

\[ \langle \bar{\psi} \psi \rangle \approx \sqrt{\frac{|eB|}{\pi}} \exp \left( \frac{\Lambda^2}{2 |eB|} \right) \exp \left( -\frac{2\pi^2}{G |eB|} \right) \]

• magnetic Catalysis in 3+1 QED

\[ m \propto \sqrt{|eB|} \exp \left( -\frac{\pi}{2} \sqrt{\frac{\pi}{2\alpha}} \right). \]
Thermo-magnetic contribution to the pressure


\[
\langle \bar{\psi} \psi \rangle = - \int \frac{d^4 p}{(2\pi)^4} \text{Tr} [iS(p)]
\]

\[
M - m = 2G \int \frac{d^4 p}{(2\pi)^4} \text{Tr} [iS(p)]
\]

Figure: The gap equation
The quark propagator in the proper time representation

\[
iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} e^{is(p_\parallel^2 - p_\perp^2 \tan(qBs) - M^2 + i\epsilon)} \\
\times \left[(\cos(qBs) + \gamma_1 \gamma_2 \sin(qBs)) (p_\parallel + M) - \frac{p_\perp}{\cos(qBs)}\right],
\]

We use the symmetric gauge

\[
A^\mu = \frac{B}{2} (0, y, x, 0)
\]

and we have defined

\[
p^\mu_\perp \equiv (0, p_1, p_2, 0), \quad p^\mu_\parallel \equiv (p_0, 0, 0, p_3), \quad p^2_\perp \equiv p_1^2 + p_2^2, \quad p^2_\parallel \equiv p_0^2 - p_3^2.
\]
After taking the trace we have

\[
\langle \bar{\psi} \psi \rangle = -4N_c N_f M \int \frac{d^4 p}{(2\pi)^4} \times \int_0^\infty ds e^{is(p_\parallel^2 - p_\perp^2 \frac{\tan(qBg)}{qBg} - M^2 + i\epsilon)}
\]

The integration over the transverse momentum gives

\[
\int \frac{d^2 p_\perp}{(2\pi)^2} e^{-i\frac{\tan(qfBg)}{qfB} p_\perp^2} = qfB \frac{1}{4\Pi i \tan(qfBg)}.
\]

In order to introduce finite temperature, we use the matsubara formalism by means of the replacement

\[
\int \frac{d^2 p_\parallel}{(2\pi)^2} \rightarrow iT \sum_n \int \frac{dp_3}{2\pi}.
\]
We also make the change of variable \( s \to i\tau \), Ending with an expression for the quark condensate

\[
\langle \bar{\psi} \psi \rangle = -N_c N_f M \frac{q_f B}{\pi} T \sum_n \int \frac{dp_3}{(2\pi)} \\
\times \int_{\tau_0}^{\infty} \frac{d\tau}{\tanh(q_f B\tau)} e^{-\tau(\tilde{\omega}_n^2 + \omega)}.
\]

with \( \tilde{\omega}_n = (2n + 1)\pi T \) and \( \omega^2 = p_3^2 + M^2 \).

after the inversion of the Jacobi Theta function, we obtain

\[
\langle \bar{\psi} \psi \rangle = -\frac{N_c N_f M q_f B}{4\pi^2} \sum_n \int \frac{dp_3}{(2\pi)} \\
\times \int_{\tau_0}^{\infty} \frac{d\tau}{\tau \tanh(q_f B\tau)} e^{-\tau M^2} \vartheta_3 \left( \frac{1}{2}, \frac{i}{4\tau \pi T^2} \right).
\]
where

\[ \vartheta \left( \frac{1}{2}, \frac{i}{4\tau\pi T^2} \right) = \sum_{n=-\infty}^{\infty} (-1)^n \exp \left( -\frac{n^2}{4\tau T^2} \right). \]

The parameter \( \tau_0 \) is fixed for \( T = 0 \) and \( B = 0 \), using a constituent mass \( M = 0.3 \) GeV and setting \(-\langle \bar{\psi}\psi \rangle = (0.269)^3 \) GeV\(^3\).

We obtain \( \tau_0 = \Lambda^{-2} = 1.54 \text{GeV}^{-2} \), and \( G_{T=0,B=0} = 7.62 \text{ GeV}^{-2} \).
We use the data from lattice to determine the parameters in the NJL model
The condensate for the quark $u$
The condensate for the quark d
The behaviour of the Coupling
The average coupling
• The pressure is given by

\[ P = -\frac{M_{ud} - m_{ud}}{4G_{ud}} - \text{Tr} \int \frac{d^4p}{(2\pi)^4} \ln \left(iS^{-1}(p)\right). \]

• We plot \( P_N = P(T, B) - P(0, 0) \).
$T_c < T$, $P_n < 0$, $G$ and the condensate increase
$T > T$, $P > 0$, $g$ and the condensate decrease.
At zero temperature, the coupling increases with B and strengthens the condensate.

The regime of catalysis and inverse magnetic catalysis are related on the behaviour of the strong coupling dependence with B.

At high temperature, the coupling decreases, implying that the condensates diminishes.

Lattice data supports our vision.