

# Wealth distribution and information flux in a differential equations model of financial market

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## Cellular automata - each automaton represents one agent

- Ying Fan, Shang-Jun Ying, Bing-Hong Wang, and Yi-Ming Wei, *Physica A* 325 (2003) 507-516
- Ying Fan, Shang-Jun Ying, Bing-Hong Wang, and Yi-Ming Wei, *Computers Industrial Engineering* 56 (2009) 63-69
- L. Bakker, W. Hare, H. Khosravi, and B. Ramadanovic, *Physica A* 389 (2010) 1223-1229
- Allbens P.F. Atman, and Bruna Amin Gonçalves, *Brazilian Journal of Physics* 42 (2012) 137-145
- Allbens P.F. Atman, and Fischer M. Stefan, *Physica A* 419 (2014) 630-641

## Differential equations - the variables represent groups of agents

- G. Caginalp, and G.B. Ermentrout, Applied Mathematics Letters 3 (1990) 17-19
- G. Caginalp, David Porter, and Vernon Smith, International Journal of Industrial Organization 18 (2000) 187-204
- G. Caginalp, and M. DeSantis, American Behavioral Scientist 55 (2011) 1014-1034

## This contribution

- Differential equations - the variables represent single agents

## Differential equations model

- Demand and supply

$$\frac{dX_j}{dt} = a_j (\log P_j - \log P) + \sum_{i \neq j} b_{j,i} X_i - c_j X_j, \quad 1 \leq j \leq N$$

- Price

$$\frac{1}{P} \frac{dP}{dt} = m \left( \frac{T_d - T_s}{T_d + T_s} \right)$$

$$\frac{d}{dt} \log P = m \left( \frac{\sum_{j=1}^N X_j}{\sum_{j=1}^N |X_j|} \right)$$

## Including liquidity limitations

- Discrete time steps:  $\Delta t$
- Agents state
  - money:  $M_j$
  - value of stocks:  $V_j$
- At each time step
  - Demand and supply limited by  $M_j$  and  $V_j$
  - Total buy equals total sell

## Including fair price variation

- Information from outside:  $P_{fair}$ 
  - At each time step  $P_{fair}$  may change
    - Probability:  $\varphi$
    - $\log P_{fair}(new) = \log P_{fair}(old) + k_1 \eta \sqrt{\Delta t} + k_2$
- Information flow
  - At each time step  $P_j$  may change to  $P_{fair}$ 
    - Probability:  $\theta D_j$



## Coefficients

- Entropic

$$S = \frac{-Q_j \log_2 Q_j}{\log_2(N)}, \quad Q_j = \frac{P_j}{\sum_{j=1}^N P_j}$$

- Price deviation

$$R = 1 - |P - P_{fair}|$$

## Parameters

$$N = 1000, \quad \Delta t = 0.01$$

$$a_j = \frac{100}{\sqrt{N-1}}, \quad c_j = 100, \quad m = \frac{10}{\sqrt{N-1}}$$

$$\varphi = 0.1, \quad \theta = 0.001, \quad k_1 = 0.1$$

$$b_{j,i} = 10 \frac{\beta_{j,i}}{\bar{D} - 1}, \quad \beta_{j,i} = 0 \text{ or } 1$$

## Initial state

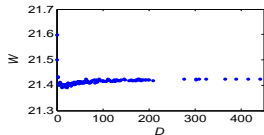
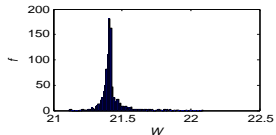
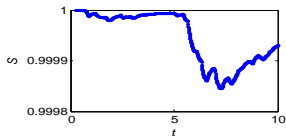
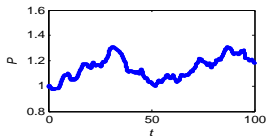
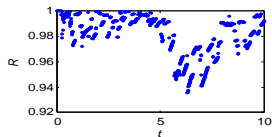
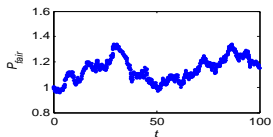
$$P = 1, \quad P_{fair} = 1$$

$$X_j = 0, \quad P_j = 1,$$

$$M_j = 10, \quad V_j = 10, \quad 1 \leq j \leq N$$

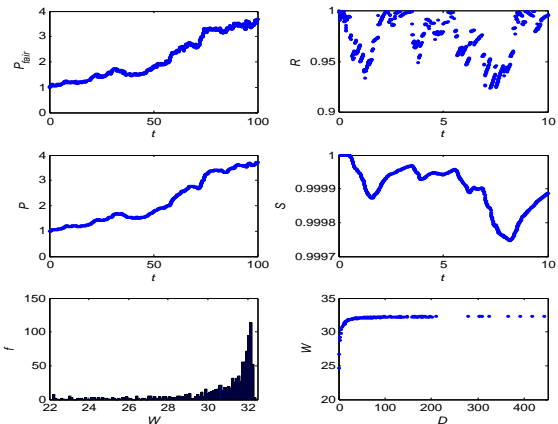
# Wealth distribution and information flux

- $\beta_{j,i}$ : preferential attachment algorithm
- $k_2 = 0$



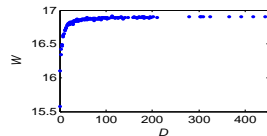
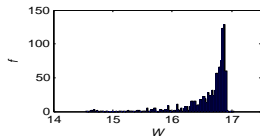
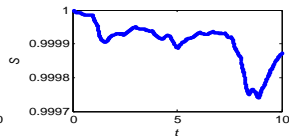
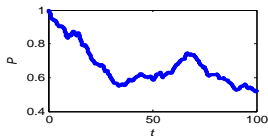
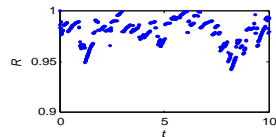
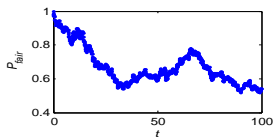
# Wealth distribution and information flux

- $\beta_{j,i}$ : preferential attachment algorithm
- $k_2 = 0.001$



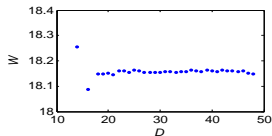
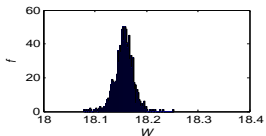
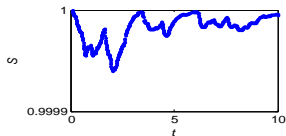
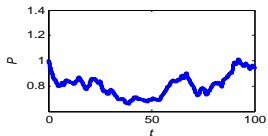
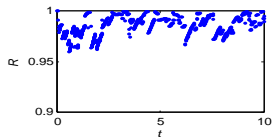
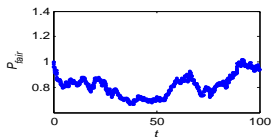
## Wealth distribution and information flux

- $\beta_{j,i}$ : preferential attachment algorithm
- $k_2 = -0.001$



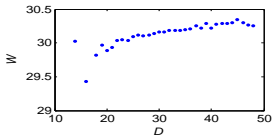
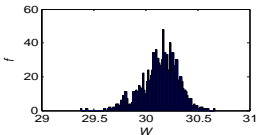
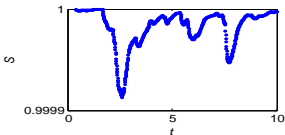
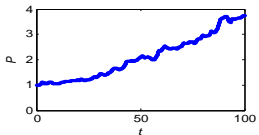
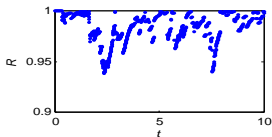
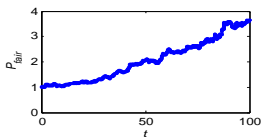
# Wealth distribution and information flux

- $\beta_{j,i}$ : Erdős-Rényi model
- $k_2 = 0$



# Wealth distribution and information flux

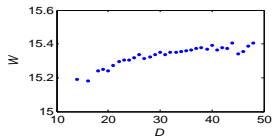
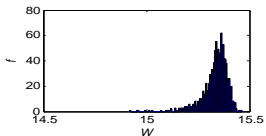
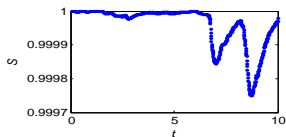
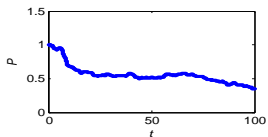
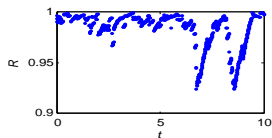
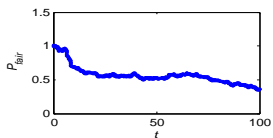
- $\beta_{j,i}$ : Erdős-Rényi model
- $k_2 = 0.001$





# Wealth distribution and information flux

- $\beta_{j,i}$ : Erdős-Rényi model
- $k_2 = -0.001$



## In this model

- Information diversity  $\leftrightarrow$  deviation from fair price
- Presence of trend
  - higher dispersion and skewness
  - advantage in getting information quickly