The Laser-Interferometric Gravitational Compass

M.D. Maia

NEW PHYSICS IN SPACE

Intitute for Theoretical Physics-ITP
South American Institute for Fundamental Research-SAIRF
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Much has been said about the technical and numerical advances of LIGO used in the recent gravitational wave event GW150914. This presentation is about its theoretical aspects. Specifically, it is about the invariant formulation of the gravitational field of General Relativity by Felix Pirani and Hermann Bondi and its implications to Gravitational waves.

The Relevant references are in arXiv:1609.06383v1 [gr-qc]
Basic Tool: The Geodesic Deviation Equation

Consider two test particles with mass at positions A and B under the influence of a gravitational field. A sends a signal to B with velocity $P$, along the shortest distance

$$\nabla_P P = 0 \text{ signal geodesic}$$

$$\nabla_T T = 0 \text{ free fall geodesic}$$

In order to close the parallelogram the signal speed must vary along the fall, such that

$$\nabla_T P = \nabla_P T \text{ closing condition}$$
The Riemann tensor calculated in the geodesic parallelogram is

\[ R(T, P)W = [\nabla_T, \nabla_P]W = \nabla_T(\nabla_P W) - \nabla_P(\nabla_T W) \]

Denoting the fall acceleration of the signal, \( a = \nabla_T P \), we obtain the geodesic deviation equation

\[ \nabla_T a = R(T, P)T \]

or, using an arbitrary basis \( \{e_\mu\} \), its components are

\[ R(e_\mu, e_\nu)e_\rho = R_{\mu\nu\rho\sigma}e_\sigma, \quad \mu = 0, 1, 2, 3 \]

In particular, taking \( T = e_0 \) and \( P = e_i \), for any \( i = 1, \) or \( 2, \) or \( 3 \)

\[ \frac{da_i}{cdt} = R_{0i0i} \]
For a gravitational wave two masses separated by a length $L$ are monitored by laser telemetry device, measured by a Fabri-Perot interferometer. The Masses are suspended to compensate for the static Earth’s gravitational pull, so that initially the space-time mimics an empty Minkowski space-time.

When they are hit by a gravitational wave, they surf on the gravitational wave on the 2-dimensional world-sheet with signature $(+, -)$. In terms of the frequency shift $a_i = ae^{i\omega_it}$,

Geodesic deviation $\frac{1}{\omega_i} \frac{d\omega_i}{dt} = R_{i0i0}$
Any 2-dimensional Riemannian geometry is conformally flat:

\[ g_{ij} = e^{2\varphi} \eta_{ij}, \]
\[ R_{ij} = (\varphi,_{11} - \varphi,_{22})\eta_{ij} \]
\[ R = 2e^{-2\varphi}(\varphi,_{11} - \varphi,_{22}) \]
\[ \Rightarrow \quad R_{ij} \equiv \frac{1}{2} R g_{ij} \]

Einstein’s equations vanish identically in 2 dimensions: \textit{General Relativity is not valid in 2-dimensional space-times.}

Consequently, any gravitational waves detected by a 2-masses laser interferometer cannot be explained by General Relativity. Possible 2-dimensional gravitational theories are the Jackiw-Teitelboim theory and the Liouvile gravity theory.
The LIGO/VIRGO/GEO laser interferometric gravitational wave detectors added a third mass (in fact they use four coplanar masses, with two of them close together at the vertex), so that the system define a plane surface in space-time.

When that plane is hit by a gravitational wave, it generates a 3-dimensional curved wavy world-volume, with signature $(+,+,−)$ whose curvature feeds the Fabri-Perrot interferometer with a superposition of frequency shifts

$$\frac{1}{\omega_{ij}} \frac{d\omega_{ij}}{dt} = R_{i0j0}, \quad i, j = 1, 2$$
This represented a substantial improvement of the Gravitational wave detection because the Einstein’s equations are valid in the 3-dimensional curved wave world-volume.

Nonetheless, in a 3-dimensional Riemannian geometry the curvature tensor is completely determined by the Ricci tensor. In particular for a 3-dimensional space-time (see eg Steven Carlip)

\[ R_{ijkl} = 2g_{i[k}R_{l]j} - 2g_{j[k}R_{l]i} - g_{i[k}g_{l]j}, \quad i, j = 0, 1, 2 \]

If we combine this with Einstein’s equations written as

\[ R_{\mu\nu} = \kappa(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}) \]

then the Riemann curvature of the world-volume will be completely determined by the local matter.
Since the local gravitational field of all known matter near the detector was annihilated (by the suspension of the masses), in theory the interference fringes of a 3-mass detector should not change.

Possible explanations for the GW150914

(a) The gravitational wave produced a change of the local topology of the world volume. One example is given by the Chern-Simmons Lagrangian

\[ \mathcal{L}_{CS} = \epsilon^{ijk} (\Gamma_{in}^m \Gamma_{km,j}^n + \frac{2}{3} \Gamma_{in}^m \Gamma_{jp}^n \Gamma_{km}^p), \quad i, j, k, \ldots = 0, 1, 2 \]

(b) Bondi’s News function resulting from the BMS symmetry produced before the coalescence of the two black holes, modified the local geometry of the world-volume.
Understanding the Invariant Theory of Gravitational Waves

The invariant theory of gravitation and of gravitational waves was introduced by F.A.E. Pirani in the 60’s. Essentially it states that the observables of Einstein’s gravitational field are given by the eigenvalues of Weyl tensor

\[ C_{\mu \nu \rho \sigma} X^{\mu \nu} = \lambda X_{\rho \sigma} \]

There are at most six independent eigenvalues and six eigenvectors, corresponding to the six Petrov types

\[ C_{ijkl} = \frac{C^O_{ijkl}}{r^5} + \frac{C^I_{ijkl}}{r^4} + \frac{C^{II/D}_{ijkl}}{r^3} + \frac{C^{III}_{ijkl}}{r^2} + \frac{C^N_{ijkl}}{r} \]

The peeling off theorem shows the decay of the Petrov types with the distance
Laser Gravitational Wave Detectors with 4 Masses

In 1957 F. Pirani proposed an experiment to measure the local gravitational field: you simply throw a handful of masses in a region of the spacetime, and measure their relative displacements. Peter Szekeres improved Pirani’s device by arguing that only 4 masses corresponding to the six eigenvalues of the curvature tensor are required. He connected these masses by six dynamometers.

The Gravitational Compass: When AB, BC, AC vanish, the remaining eigenvectors point to the direction of the source.
The Pirani/Szekeres Laser Gravitational Compass offers a true measurement of the relativistic gravitational waves, by combining the two ideas: The use of only 4 masses, together with a Laser telemetry to measure the displacements in place of the Szekeres dynamometers.

Since four points define a sphere in a 3-dimensional space, the motion of the four masses correspond to the principal modes of oscillation the spherical world-volume.

P-modes of Oscillations in a Neutron Star
Constructing a Laser Gravitational Compass

The construction of a 4 masses gravitational wave detector can be obtained as an upgrade of some currently existing laser interferometers with 4 masses, by adding a tower with a mass + mirror at the top. A 300 mts gravitational compass prototype, with 10 kilogram masses is feasible. The swing of the tower can be minimized with the use of multiple stays. Inside the tower a steel tube will concentrate the laser rays and cooling media. Three Fabri-Perot interferometers may be required to measure the 3-dimensional data of the gravitational radiation.

The Gravitational Compass for GW detection
A 300 meters ecological tower in the Amazon shows the feasibility of such prototype where the same advanced Laser technology of the GEO600 (Germany) can be used.

We contemplate the possibility of a Brazilian (INPE)-German(AEI) agreement for gravitational astronomy, with a 300m tower in Brazil and a 600 meter tower in Germany.
Conclusions/Perspectives

Due to geometric limitations, the 2 and 3-mass Laser based detectors are unable to detect the full range of gravitational observables defined in four-dimensional space-times. These limitations can be eliminated only with the inclusion of the five effective eigenvalues of the gravitational field with the addition of a fourth mass. Since all observables of the gravitational field are present, the sensitivity of such detectors will be necessarily greater than the 3 masses ones, including pointing out the direction of the sources.