Hawking-like radiation model for inflation and baryogenesis*

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Outline of Talk

• Particle Creation in Cosmology

• Modified FRW Equations and Consequences

• Gravitational Baryogenesis

• Summary
Inflation basic details

- Rapid expansion in the early Universe addresses horizon, flatness, monopole/heavy relic problem, and gives a causal mechanism for seeds leading to the LSS.

- Questions: how does inflation start/stop? What is mechanism for inflation?

![Graph showing expansion of the observable universe](image)
Influence of particle production/decay on cosmology

• Idea that particle production influences cosmological evolution goes back to Erwin Schrödinger [Physica 6, 899 (1930)] and Parker [PRL/PR 1968/1969].

• Our approach is similar to Prigogine, et al. in [GRG 21, 767 (1989)]. See also works by JAS Lima et al. [PLA 1992, PRD 1995 and onward].

• Prigogine et al. considered generic particle creation. We consider Hawking-like radiation as the particle creation mechanism.
Advantage of generic particle creation inflation

• Prigogine *et al* emphasized two advantages of their particle creation model of inflation:

1. Explains the enormous entropy production in the early Universe via the *irreversible energy flow from the gravitational field to the created particles*.

2. Initial singularity is avoided. The Universe begins from an *instability of the vacuum* instead of a singularity.
FRW equations for matter creation

• In the presence of matter creation the equations of the standard FRW metric become [JAS Lima et al. PRD 1995]

\[ 3 \frac{\ddot{a}}{a^2} = \frac{8\pi G \rho}{c^2} \]

\[ 2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a} = -\frac{8\pi G}{c^2}(p + p_c) \]

\[ \frac{\dot{n}}{n} + 3 \frac{\dot{a}}{a} = \frac{\psi}{n} = \Gamma \]

\[ p_c = -\frac{\rho + p}{3H} \Gamma \]

\[ H = \frac{\dot{a}}{a}. \]

• 1\textsuperscript{st} Friedmann equation unaltered; 2\textsuperscript{nd} Friedmann equation has creation pressure term; 3\textsuperscript{rd}/new equation is particle number density \((n)\) and \(\Gamma = \psi/n = \text{generic creation rate per number density.}\)
FRW equations for matter creation

- A general phenomenological creation rate per particle density can be written as
  \[ \frac{\Gamma}{3H} = \alpha + \beta \left( \frac{H}{H_i} \right) + \gamma \left( \frac{H}{H_i} \right)^2 + \ldots \]

- Where \( \alpha, \beta, \gamma \) are dimensionless constants and \( H_i \) is some scale.

- FRW emits Hawking-like radiation [T. Zhu et al, IJMPD 19, 159 (2010)] with temperature \( \rightarrow \) particle creation.

- This gives a Hawking-like temperature which can be converted into a creation rate.
  \[ T \approx \frac{\hbar H}{2\pi k_B} \quad \Rightarrow \quad \frac{\Gamma}{3H} = \frac{1}{15} \left( \frac{H}{H_{Pl}} \right)^2 \]

  \[ H_{Pl} = \frac{1}{t_{Pl}} \]
Thermodynamics and Particle Creation

- This particle creation modifies FRW continuity equation

\[ \frac{\dot{\rho}}{\rho} + 3(1 + \omega) \frac{\dot{a}}{a} = 3\omega_c(t) \frac{\dot{a}}{a} \]

\[ \omega_c(t) = \alpha \rho(t) \]

\[ \alpha = \frac{\hbar G^2}{45c^7} \]

- Setting \( \omega = 1/3 \) (relativistic matter) and solving the above gives

\[ \rho = \frac{D_0 a^{-3(1+\omega)}}{1 + \left( \frac{\alpha D_0}{1+\omega} \right) a^{-3(1+\omega)}} \rightarrow \frac{D_0}{a^4 + \frac{3\alpha D_0}{4}} \]
Very early Universe limit

- For very early universe one has $\alpha D_0 >> a^4$ and $\rho \sim 4 \alpha / 3$ one has an effective dS solution supported by radiation.

- The *inflationary* solution for $a(t)$ is now ($K_0$ is integration constant)

\[
a(t) = 2(3\alpha D_0)^{1/4} \exp \left[ \sqrt{\frac{32\pi G}{9c^2\alpha}} t - \frac{K_0}{2} \right]
\]

- The Hubble constant is enormous

\[
H = \frac{\dot{a}}{a} = \sqrt{\frac{32\pi G}{9c^2\alpha}} \approx 10^{45} \frac{1}{\text{sec}}
\]
Very early Universe limit

- Exponential expansion
• For early universe one has $a^4 \gg \alpha D_0$ and $\rho \sim D_0 / a^4$

• The solution now is that of radiation

$$a(t) \approx \left( \frac{32\pi GD_0}{3c^2} \right)^{1/4} t^{1/2}$$

• This gives a natural transition from $\text{Exp}[..]$ expansion to radiation power law expansion.
Exit from inflation

- Time is in Planck time and different values of $K_0$. Transition from $a(t) \sim \text{Exp} [...]$ to $a(t) \sim t^{1/2}$ behavior.
• Here inflation is driven by Hawking radiation/particle creation. *Reverse of BH evaporation.*

• It is speculated that in the quantum gravity regime Hawking radiation turns off [e.g. *P. Nicolini, IJMPA 24, 1229 (2010)*].

(i) The Universe expands according to $t^{1/2}$ until one exits the QG regime
(ii) Hawking-like radiation turns on and drives inflation until the Hawking temperature drops below a critical value
(iii) At which point the expansion becomes $t^{1/2}$ again.
## Comparison with standard inflation

<table>
<thead>
<tr>
<th>Standard Inflation</th>
<th>Inflation via Particle Creation</th>
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</thead>
<tbody>
<tr>
<td>Size increase by a factor of $10^{26}$</td>
<td>Size increase by a factor of $\sim 10^{26}$</td>
</tr>
<tr>
<td>$a(t)$ goes from $\sim 10^{-27}$ m to $\sim 10^{-1}$ m</td>
<td>$a(t)$ goes from $\sim 10^{-32}$ m to $\sim 10^{-6}$ m</td>
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<tr>
<td>Lasts for a time $\Delta t \sim 10^{-32}$ to $10^{-33}$ sec</td>
<td>Lasts for a time $\Delta t \sim 10^{-43}$ sec</td>
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<tr>
<td>Scale of inflation $\leq 10^{16}$ GeV</td>
<td>Scale of inflation around Planck scale</td>
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</table>
Comparison with standard inflation

• Some aspects incompatible with ideas of standard inflation.

• Some “nice” features – no scalar field, turn-on/turn-off mechanism.

• Possible fix:

The dimensionality of the early Universe is different so \( G \) at present is an effective \( G \) different from \( G_0 > G \) in the early Universe [\textit{ADD-scenario PLB 1998 or Mureika Stojkovic PRL 2011}]. Makes \( \alpha \) larger.
Baryogenesis

• The Universe contains more matter than anti-matter

• Characterized by

\[ \eta = \frac{n_B - n_B}{s} \approx 6 \times 10^{-10} \]

• Sakharov gave three conditions for baryogenesis
  (i) Violation of baryon number
  (ii) C and CP violation
  (iii) Departure from thermal equilibrium

• How to explain baryogenesis in gravitationally induced particle production models?
Gravitational baryogenesis

• A string theory inspired gravitational B-L interaction [H. Davoudiasl et al PRL (2004)]

\[
\frac{\hbar^3}{M_*^2 c} \int d^4x \sqrt{-g} (\partial_\mu \mathcal{R}) J^\mu_{B-L}
\]

• Allows on to define a chemical potential which distinguishes particle species (charge \(q_i \to \pm 1\)) based on B-L charge

\[
\mu_i = \frac{q_i \hbar^3}{c^2 M_*^2} \mathcal{R} \rightarrow \pm \frac{\hbar^3}{c^2 M_*^2} \mathcal{R}
\]
Gravitational baryogenesis

- In a thermal bath the rate per area of B-L creation is [in A. Hook PRD (2014) primordial BHs]

\[
\frac{n_B - n_{\bar{B}}}{1} = \int \frac{d^3 p}{(2\pi \hbar)^3} \left( \frac{1}{e^{(pc-\mu)/k_BT} + 1} \right) - \int \frac{d^3 p}{(2\pi \hbar)^3} \left( \frac{1}{e^{(pc+\mu)/k_BT} + 1} \right)
\]

- Above has Pauli-Dirac statistics since in SM only fermions carry B and L numbers

- Integrating the above and assuming \( \mu \ll T \) gives

\[
\frac{n_B - n_{\bar{B}}}{1} \approx \frac{\mu k_B^2 T^2}{6(\hbar c)^3}
\]
Gravitational baryogenesis (entropy density)

• The entropy density is given by

\[ s = \frac{2\pi^2}{45(\hbar c)^3} g_* (k_B T)^3 \]

• Which with the previous result gives the baryon asymmetry parameter

\[ \eta = \frac{n_B - n_{\bar{B}}}{s} \approx \frac{15\mu}{4\pi^2 g_* k_B T} \]

• Where \( g^* \sim 100 \) is the total number of (SM) degrees of freedom.
Gravitational baryogenesis: chemical potential

- To get the chemical potential need change in Ricci scalar. At tree level

\[ \dot{R} = 9(1 - 3\omega)(1 + \omega) \frac{H^3}{c^2} \]

- This is zero for \( \omega = 1/3 \), -1 (radiation and dS)

- To one-loop (g=SU(3) coupling, \( N_c = 3 \), \( N_F = 6 \)) \([Kajantie, et al., PRD (2003)]\)

\[
1 - 3\omega = \frac{5}{6\pi^2} \frac{g^4}{(4\pi\hbar c)^2} \frac{(N_c + \frac{5}{4} N_f)(\frac{11}{3} N_c - \frac{2}{3} N_f)}{2 + \frac{7}{2}[N_c N_f/(N_c^2 - 1)]} \approx 10^{-2} - 10^{-3}
\]
Gravitational baryogenesis: chemical potential

• The full chemical potential assuming $\omega \sim 1/3$ is thus

$$
\mu \approx \frac{\hbar^3 g_{*B} H^3}{10c^4 M_*^2}
$$

• Which gives for the baryon asymmetry

$$
\eta \approx \frac{\hbar^2 H^2}{100c^4 M_*^2} \approx \frac{\hbar^2}{400c^4 t^2 M_*^2}
$$

• To get the observed $\eta \sim 6 \times 10^{-10}$ we need to take $t \sim 10^{-39}$ sec which can be accomplish by setting $K_0 \sim 10^6$
Conclusions

• Generic particle creation (a la Prigogine et al.) can drive inflationary expansion. Here generic particle creation $\rightarrow$ Hawking-like radiation.

• Explains large entropy production and avoids initial singularity.

• Gives a natural exit and (maybe) entrance to inflation.

• Same mechanism can give rise to baryogenesis.

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Scale factor $a(t)$

- For given $\rho(t)$ and $\Gamma(t)$ one can find an exact, analytical expression for $a(t)$

\[
\sqrt{\alpha D_0 + \frac{4}{3}a^4 + \sqrt{\alpha D_0} \ln \left[ \frac{a^2}{2 \sqrt{3} \left( \sqrt{\alpha D_0} + \sqrt{\alpha D_0} + \frac{4}{3}a^4 \right)} \right]} = \frac{8}{3} \sqrt{\frac{2\pi GD}{c^2}} t - (K_0 - 1) \sqrt{\alpha D_0}
\]
Thermodynamics and Particle Creation

- From the first law of thermodynamics

\[ \frac{dQ}{dt} = \frac{d}{dt}(\rho V) + p \frac{dV}{dt} \]

- Usual assumption – universe is closed, adiabatic system \(dQ=0\). Hard to explain large entropy production \(TdS=dQ=0\).

- Due to Hawking radiation of FRW space-time we take from the Stephan-Boltzmann Law \((A_H\) is horizon area)

\[ P = + \frac{dQ}{dt} = \sigma A_H T^4 \]

\[ \sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2} \]
Thermodynamics and Particle Creation

• For the area we use effective horizon radius

\[ \text{Area} = 4\pi r_{\text{FRW}}^2 = \frac{4\pi c^2}{H^2} \]

• For the temperature we take FRW temperature

\[ T \approx \frac{\hbar H}{2\pi k_B} \]
Thermodynamics and Particle Creation

• Using the above results we find

\[
\frac{d}{dt}(\rho V) + p\frac{dV}{dt} = \sigma A_H \left( \frac{\hbar H}{2\pi k_B} \right)^4
\]

• We now find

\[
\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = \frac{3\sigma}{c} \left( \frac{\hbar}{2\pi k_B} \right)^4 H^5.
\]

• Leading to

\[
\frac{\dot{\rho}}{\rho} + 3(1 + \omega)\frac{\dot{a}}{a} = 3\omega_c(t)\frac{\dot{a}}{a}
\]
Thermodynamics and Particle Creation

- Particle creation actually changes the simple expression for $\frac{dR}{dt}$

$$\dot{R} = 9 \left[ (1 - 3\omega) + (1 + \omega) \frac{\Gamma}{H} \right] \left[ 1 - \frac{\Gamma}{3H} \right] (1 + \omega) H^3 - 3H^2(1 + \omega) \frac{d}{dt} \left( \frac{\Gamma}{H} \right)$$

- Thus the particle production itself shifts the chemical potential w/o the need for QCD loop corrections. [Work on going].

- Assuming $\omega = 1/3$ and

$$\frac{\Gamma}{3H} \approx \left( \frac{H}{H_I} \right)^n$$

- We find

$$\dot{R} = 24 \frac{H^{n+3}}{H_I^n} \left[ 1 - \frac{H^n}{H_I^n} \right] (2 + n)$$