Prime numbers, Riemann zeros and Quantum field theory

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Introduction

Definition 1:

A natural number $a > 1$ is called a prime number if it has only two positive divisors (namely 1 and a).

Theorem (1. Every natural number $a > 1$ can be represented as a product of prime numbers:)

$$a = \prod_{n=1}^{r} p_n^{\alpha_n} = p_1^{\alpha_1} p_2^{\alpha_2} ... p_r^{\alpha_r}, \quad r \geq 1. \tag{1}$$

Definition 2:

Every number > 1 which is not a prime is called a composite number. The natural numbers fall in three classes:

1. the number 1;
2. the primes;
3. the composite numbers.

Theorem (2. There are infinitely many primes.)

Proof: Let $p_1, p_2, ..., p_r$ be distinct prime numbers. Then

$$a = 1 + \prod_{n=1}^{r} p_n. \tag{2}$$

By Theorem 1, a is divisible by a prime number different from $p_1, \cdots p_r$.

Prime Numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Prime Numbers

- Prime numbers occur in a very irregular way within the sequences of natural numbers. The best result that we have concerning their global distribution is the prime number theorem:

$$\pi(x) \sim \frac{x}{\ln x}. \qquad (3)$$

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- The Riemann hypothesis claims that all nontrivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\Re(s) = 1/2$.
- There is a connection between the Riemann zeros and the distribution of prime numbers.
- In the region of the complex plane where $\zeta(s)$ converges absolutely and uniformly there is a representation in terms of the product of all prime numbers.

David Hilbert: Mathematical Problems (1900).

8th: Problem of Prime Numbers

- Hilbert and Pólya suggested that there might be a spectral interpretation for the non-trivial zeros of the Riemann zeta function. The nontrivial zeros could be the eigenvalues of a self-adjoint linear operator in an appropriate Hilbert space. The existence of such operator may lead to the proof of the Riemann hypothesis.

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- The two-point correlation function of the zeros of the zeta function in the critical line is equal to the two-point correlation function of the eingenvalues of a random Hermitian matrix taken from the Gaussian unitary ensemble.

- Quantum mechanics show us how to represent physical systems, how to represent observations, how to carry measurements and how systems evolve when not measured.

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- Quantum field theory is the formalism describing systems with infinitely many degrees of freedom using the probabilistic interpretation of quantum mechanics and the special theory of relativity.
- This framework enables one to exploit questions related to number theory using statistical mechanics methods. Such a program is called the arithmetization of quantum field theory.

The Riemann zeta function

Let s be a complex variable i.e. $s = \sigma + i\tau$ with $\sigma, \tau \in \mathcal{R}$. For $\Re(s) > 1$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{1}{1 - p^{-s}} \right) \tag{4}$$

where p is the sequence of the prime numbers.
Proof:

$$E_p(s) = \frac{1}{1 - p^{-s}} = 1 + \frac{1}{p^s} + \frac{1}{p^{2s}} + \dots \tag{5}$$

$$\prod_{i=1}^{N} E_{p_i}(s) = \left(1 + \frac{1}{p_1^s} + \frac{1}{p_1^{2s}} + \dots \right) \left(1 + \frac{1}{p_2^s} + \frac{1}{p_2^{2s}} + \dots \right) \dots \tag{6}$$

The analytic continuation of Riemann zeta function.

$$\zeta(s) = \frac{1}{\Gamma(\frac{s}{2})} \int_0^\infty dx\, \pi^{\frac{s}{2}}\, x^{\frac{s}{2}-1} \sum_{n=1}^\infty e^{-n^2\,\pi\, x}. \tag{7}$$

Defining the function $\psi(x)$ as

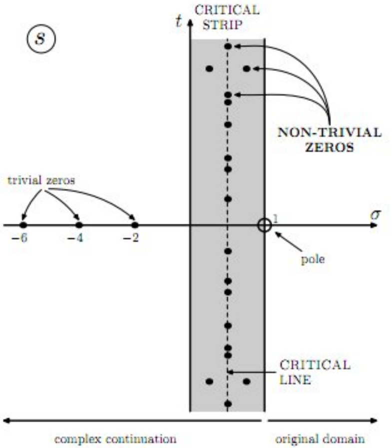
$$\psi(x) = \sum_{n=1}^\infty e^{-n^2\,\pi\, x}, \tag{8}$$

we have

$$\Gamma\left(\frac{s}{2}\right)\pi^{-\frac{s}{2}}\,\zeta(s) = \frac{1}{s(s-1)} + \int_1^\infty dx\,\psi(x)\left(x^{\frac{s}{2}-1} + x^{-\frac{1}{2}(s+1)}\right). \tag{9}$$

It gives the analytic continuation of the Riemmann zeta function to the whole complex s-plane. The only singularity being the pole at $s=1$.

Riemann zeta function on the complex s-plane



The partition function for the harmonic oscillator

We start defining the Euclidean time $\tau = it$, and consider a one-dimensional quantum mechanical system.

The partition function for the system in thermal equilibrium with a reservoir at temperature β^{-1} is

$$Z_\beta = \int_{x(0)=x(\tau)} [dx(\tau)] \ \exp\left[-\int_0^\beta d\tau \left(\frac{1}{2}m\left(\frac{dx}{d\tau}\right)^2 + V(x(\tau))\right)\right], \tag{10}$$

where in the functional integral we require that $x(\tau)$ is periodic with period β

$$x(\tau) = x(\tau + \beta). \tag{11}$$

We are using $\int_\beta [dx(\tau)]$ in the functional integral to remember that $x(\tau)$ is periodic with period β, i.e., $x(\tau) = x(\tau + \beta)$.

$$Z_\beta = \int_\beta [dx(\tau)] \; \exp\left[-\int_0^\beta d\tau \, \frac{1}{2} x(\tau) \left(-\frac{d^2}{d\tau^2} + \omega^2\right) x(\tau)\right]. \qquad (12)$$

It is a standard procedure to define the following kernel $K(\omega; \tau - \tau')$ by the equation

$$K(\omega; \tau - \tau') = \left(-\frac{d^2}{d\tau^2} + \omega^2\right) \delta(\tau - \tau'). \qquad (13)$$

Using the kernel the partition function Z_β becomes

$$Z_\beta = \int_\beta [dx(\tau)] \ \exp\left[-\int_0^\beta d\tau \int_0^\beta d\tau' \, \frac{1}{2} x(\tau) K(\omega; \tau - \tau') x(\tau')\right]. \quad (14)$$

The generating functional $Z_\beta(h)$ is defined by

$$Z_\beta(h) = \int_\beta [dx(\tau)] \exp\left[-\int_0^\beta d\tau \int_0^\beta d\tau' \frac{1}{2} x(\tau) K \, x(\tau') + \int_0^\beta d\tau \, h(\tau) x(\tau)\right]. \quad (15)$$

Since the integrations above are Gaussian it is straightforward to write

$$Z_\beta(h) = Z_\beta \, \exp\left[- \int_0^\beta d\tau \int_0^\beta d\tau' \, \frac{1}{2} h(\tau) G(\omega; \tau - \tau') h(\tau') \right], \qquad (16)$$

where the partition function is defined by $Z_\beta = Z_\beta(h)|_{h=0}$, and the Green function $G(\omega; \tau - \tau')$ is the inverse kernel, defined by

$$\int_0^\beta d\tau' \, K(\omega, \tau - \tau') G(\omega, \tau' - \tau'') = \delta(\tau - \tau''). \qquad (17)$$

.

At this point it is important to note that D_β defined by the equation

$$D_\beta = \left(-\frac{d^2}{d\tau^2} + \omega^2\right), \tag{18}$$

is a positive definite elliptic operator acting on $x_n(\tau)$, defined in a compact manifold. The operator D_β has a complete set of orthonormal eigenfunctions $x_n(\tau)$ with eigenvalues λ_n.

$$\left(-\frac{d^2}{d\tau^2} + \omega^2\right) x_n = \lambda_n\, x_n, \tag{19}$$

with the periodic boundary condition $x_n(0) = x_n(\beta)$. Explicitly we have $\lambda_n = (\frac{2\pi n}{\beta})^2 + \omega^2$. The partition function is given by $Z_\beta(h = 0) = (\det\, D_\beta)^{-1/2}$.

The spectral zeta function for the operator D_β is

$$\zeta_D(s) = \sum_n \frac{1}{\lambda_n^s}. \tag{20}$$

It is clear that

$$\ln \det D_\beta = -\frac{d}{ds}\zeta_D(s)|_{s=0}, \tag{21}$$

where the derivative of the zeta function has to be determined by analytic continuation from the domain where the defining series actually converges. Therefore $\ln Z_\beta$ can be written as

$$\ln Z_\beta(h=0) = \frac{1}{2}\frac{d}{ds}\zeta_D(s)|_{s=0} \tag{22}$$

To proceed let us substitute the explicit expression for λ_n in the spectral zeta function associated with the operator D_β.

We have

$$\zeta_D(s) = \sum_{n=-\infty}^{\infty} \left[\left(\frac{2\pi n}{\beta} \right)^2 + \omega^2 \right]^{-s}. \tag{23}$$

The series converges for $\Re(s) > \frac{1}{2}$ and its analytic continuation defines a meromorphic function of s, regular at $s = 0$.

$$\ln Z_\beta(h=0) = \frac{1}{2} \frac{d}{ds} \zeta_D(s)|_{s=0} = -\frac{1}{2}\beta\omega - \ln(1 - e^{-\beta\omega}). \tag{24}$$

Free energy: $F_\beta = -\frac{1}{\beta} \ln \, Z(\beta, h)|_{\, h=0}$

Mean energy $E(\beta) = -\frac{\partial}{\partial\beta} \ln Z(\beta, h)|_{\, h=0}$,

Euclidean field theory

Let us consider a neutral scalar field with a $(\lambda\varphi^4)$ self-interaction, defined in a d-dimensional Minkowski spacetime. The vacuum persistence functional is the generating functional of all vacuum expectation value of time-ordered products of the theory.

The Euclidean field theory can be obtained by analytic continuation to imaginary time.

In the Euclidean field theory, we have the generating functional of complete Schwinger functions. The $(\lambda\varphi^4)_d$ Euclidean theory is defined by these Euclidean Green's functions.

The Euclidean generating functional $Z(h)$ is formally defined by the functional integral:

$$Z(h) = \int [d\varphi] \; \exp\left(-S_0 - S_I + \int d^d x \, h(x)\varphi(x) \right), \tag{25}$$

where the action that describes a free scalar field is

$$S_0(\varphi) = \int d^d x \left(\frac{1}{2}(\partial\varphi)^2 + \frac{1}{2} m_0^2 \, \varphi^2(x) \right), \tag{26}$$

and the interacting part is

$$S_I(\varphi) = \int d^d x \, \frac{\lambda}{4!} \, \varphi^4(x). \tag{27}$$

In Eq.(25), $[d\varphi]$ is formally given by $[d\varphi] = \prod_x d\varphi(x)$. The terms λ and m_0^2 are respectively the bare coupling constant and mass squared of the model.

The prime zeta function

To proceed, we have the prime zeta function $\zeta_p(s)$, $s = \sigma + \tau i$, for real σ and τ. We have

$$\zeta_p(s) = \sum_{\{p\}} p^{-s}, \ \Re(s) > 1, \tag{28}$$

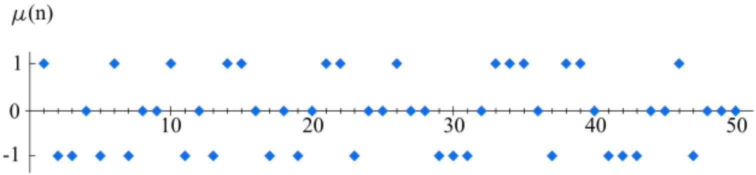
where the summation performed over all primes. The series converges absolutely when $\sigma > 1$.

The Möbius function $\mu(n)$:

$$\mu(n) = \begin{cases} 1, & \text{if n=1,} \\ (-1)^r, & \text{if n is the product of r } (\geq 1) \text{ distinct primes,} \\ 0 & \text{otherwise, i.e., if the square of at least} \\ & \text{one prime divides n.} \end{cases}$$

The Möebius function

$$\mu(1) = 1,\ \mu(2) = -1,\ \mu(3) = -1,\ \mu(4) = 0,\ \mu(5) = -1,\ \mu(6) = 1.$$



Let us study the analytic extension of the prime zeta function. Using the Euler formula we have

$$\ln\zeta(s) = \sum_{\{p\}}\sum_{r=1}^{\infty}\frac{1}{r}\,p^{-rs}, \quad \Re(s) > 1. \tag{29}$$

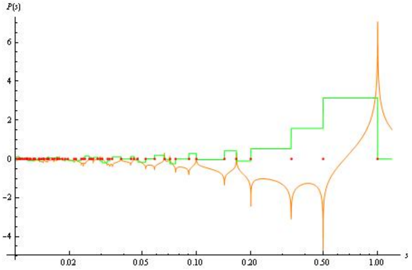
Using the definition of the prime zeta function we have

$$\ln\zeta(s) = \sum_{r=1}^{\infty}\frac{1}{r}\,\zeta_p(rs), \quad \Re(s) > 1. \tag{30}$$

Using the definition of the Möbius function $\mu(n)$, it is possible to show that the prime zeta function $\zeta_p(s)$ can be expressed as

$$\zeta_p(s) = \sum_{k=1}^{\infty}\frac{\mu(k)}{k}\,\ln\zeta(ks), \quad \Re(s) > 1. \tag{31}$$

The Riemann zeta function $\zeta(s)$ has a pole at $s = 1$. This relationship shows that $s = \frac{1}{k}$ is a singular point for all square free positive integers k. This sequence limits to $s = 0$.



There is a clustering of singular points along the imaginary axis emanating from the Riemann zeros. Therefore the line $\Re(s) = 0$ is a natural boundary of $\zeta_p(s)$.

Theorem (Landau and Walfisz (1919))

The prime zeta function has an analytic continuation only in the strip $0 < \sigma \leq 1$.

Theorem (Seeley (1967), Dowker and Crithley (1976), Hawking (1977))

$\prod_k \lambda_k = \exp\left(-\zeta_D^{'}(0)\right) \qquad \zeta_D(s) = \sum_{k=0}^{\infty} \frac{1}{\lambda_k^s} \qquad \Re(s) > s_0$

Theorem (Menezes and Svaiter (2013))

There is no physical system described by a field theory with prime numbers as its spectrum.

Riemann zeta zeros and quantum field theory

The Riemann zeta function $\zeta(s)$ defined by analytic continuation of a Dirichlet series has a simple pole with residue 1 at $s = 1$, trivial zeros at $s = -2n$, $n = 1, 2, ...$ and infinitely many complex zeros $\rho = \beta + i\gamma$ for $\beta, \gamma \in \mathbb{R}$ and $0 < \beta < 1$.

Super-zeta functions or secondary zeta functions:

$$G_\gamma(s) = \sum_{\gamma>0} \gamma^{-s}, \ \ \Re(s) > 1, \tag{32}$$

Using regularity property of one of this function at the origin, we show that it is possible to extend the Hilbert-Pólya conjecture to systems with countably infinite number of degrees of freedom.

Riemann zeros in the spectrum of vacuum modes

The eigenfrequencies of the vacuum modes are

$$\left(-\Delta_{d-1} - L\right)\varphi(\mathbf{x}_\perp, z) = \omega^2\varphi(\mathbf{x}_\perp, z). \tag{33}$$

The linear operator L has a differential and a background contribution:

$$-L = \left(-\mathcal{O}_z + \sigma(z)\right), \tag{34}$$

where $\mathcal{O}_z$ is an unknown differential operator. The eigenvalues of the L operator are the imaginary part of the Riemann zeta zeros. Therefore $-L$ satisfies

$$\left(-\mathcal{O}_z + \sigma(z)\right)u_n(z) = \frac{\gamma_n}{a^2}u_n(z), \tag{35}$$

where $u_n(z)$ is a countable infinite set of eigenfunctions, and a is the size of the compact spatial dimension.

The zero-point energy of an massive scalar field defined in a $(d+1)$-dimensional flat space-time is given by

$$\langle 0|H|0\rangle = \frac{1}{2}\sum_{\mathbf{k}}^{\infty}\omega_{\mathbf{k}}. \tag{36}$$

The eigenfrequencies of the vacuum modes are given by

$$\omega_{\mathbf{k}} = \sqrt{k_1^2 + k_2^2 + \cdots + k_{d-1}^2 + k_d^2 + m^2}. \tag{37}$$

where

$$k_d = \frac{\sqrt{\gamma_n}}{a} \quad n = 1, 2, \cdots, \tag{38}$$

This sum is divergent because all the vacuum modes give contribution to the zero-point energy.

The vacuum-energy per unit area $\varepsilon_{d+1}(a)$ can be written as

$$\varepsilon_{d+1}(a,m) = \frac{f(d)}{a^d}\Gamma(-d/2)\sum_{n=1}^{\infty}\left(\gamma_n + a^2 m^2\right)^{\frac{d}{2}}. \qquad (39)$$

Menezes, Svaiter and Svaiter (2013)

The Riemann zeros can be interpreted as the spectrum of a self-adjoint operator in a system described by field theory.

Dueñas and Svaiter (2014)

The renormalized zero-point energy of a massive scalar field with the Riemann zeros as the spectrum of the vacuum modes is finite.

The randomized Riemann gas

The Hamiltonian for a non-interacting bosonic field theory

$$H = \omega \sum_{k=1}^{\infty} \ln(p_k) b_k^{\dagger} b_k, \tag{40}$$

where $b_k^{\dagger}$ and b_k creation and annihilation operators and the p_k are the sequence of prime numbers.

The partition function Z is exactly the Riemann zeta function $\zeta(\beta\omega)$.

The average free energy density for an ensemble of infinite systems which is denumerable is

$$\langle f(\beta) \rangle = -\frac{1}{\beta V} \sum_{k} P(\omega_k) \ln \zeta(\omega_k \beta), \tag{41}$$

where $P(\omega_k)$ is a given discrete distribution function.

Average free energy per unit volume and mean energy density in the continuous case

$$\langle f(\beta,\lambda)\rangle = -\frac{1}{\beta V}\int d\omega\, P(\omega,\lambda)\ln\zeta(\omega\beta) \tag{42}$$

$$\langle \varepsilon(\beta,\lambda)\rangle = -\frac{1}{V}\int d\omega\, P(\omega,\lambda)\frac{\partial}{\partial\beta}\ln\zeta(\omega\beta), \tag{43}$$

$$\frac{\zeta'}{\zeta}(s) = C_1 - \frac{1}{s-1} + \sum_{\rho}\frac{1}{(s-\rho)} + \sum_{\rho}\frac{1}{\rho} + \sum_{n=1}^{\infty}\frac{1}{(s+2n)} - \sum_{n=1}^{\infty}\frac{1}{2n}, \tag{44}$$

where $C_1 = -1 - \frac{\zeta'(0)}{\zeta(0)}$ is an absolute constant, and ρ is the set of the nontrivial zeros of the Riemann zeta function.

$$\langle f(\beta) \rangle = -\frac{1}{\beta V} \sum_k P(\omega_k) \ln \zeta(\omega_k \beta), \qquad \zeta_p(s) = \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \ln \zeta(ks).$$

Fisher Zeros

Singularities of the free energy corresponding to the zeros of the partition function in the complex β plane.

Prime zeta function and average energy density

The average free energy density has a similar singularity structure in the complex β plane as the prime zeta function in the complex s plane.

Conclusions and perspectives

- The impossibility of extended the definition the analytic function $\zeta_p(s)$ to the half-plane $\sigma \leq 0$, means that quantum field theory, free or interacting with prime numbers spectrum does not exist.
- The sequence of nontrivial zeros of the Riemann zeta function is zeta regularizable. Therefore systems with countably infinite number of degrees of freedom described by self-adjoint operators whose spectra is given by this sequence admit a functional-integral formulation.
- Is it possible to explore a different point of view to shed some light in this problem?
- Is it possible to understand the behavior of the Riemann zeros in terms of the correlation functions of some quantum field theory?

Leonhard Euler (1707 -1783)

Bernhard Riemann (1826 - 1866)

"Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind never penetrate" Euler (1770).

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The End